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ORIGINAL ARTICLE

Comparison of mixed distribution with EV1 and GEV components for analyzing hydrologic data containing outlier

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Abstract An outlier is a very large or very small value that does not follow the general trend of a given data set. Outliers in rainfall data cause uncertainty in water engineering studies and estimated design events. As such, an additional mathematical tool for dealing with outliers is needed. One of the main issues in hydrologic frequency analysis is the problem of mixed distributions or multiple populations in hydrologic time series. Univariate probability distributions are unsuitable for data sets with outliers, therefore three mixed distributions (mixed Gumbel, mixed GEV, EV1–GEV) were used in this paper. The mixed Gumbel distribution was found to be the best distribution to fit to the 24-h annual maximum rainfall data at all of the rainfall gauging stations used in this study, on the basis of the minimum standard error of fit.

Keywords Design rainfall · Outliers · Hydrologic data · Mixed distributions · Standard error of fit

Introduction

The objective of rainfall frequency analysis is to estimate the design rainfall magnitude corresponding to any return period of occurrence through the use of appropriate probability distributions. The selection of an appropriate probability distribution is based on statistical tests for extreme hydrologic data in a specific region. Design rainfall magnitude, the amount of rainfall during a given duration (measured in hours) and return period (measured in years), is required in hydrologic structure design and in establishing effective water resource management plans. To estimate design rainfall, a conventional univariate frequency analysis can be applied to the annual maximum rainfall depth extracted from historical rainfall records of analogous storm durations (Chow et al. 1988).

Hydrologic time series often contain outliers, which might have serious effects on hydrologic modeling. Statistical frequency analysis is used to determine the design level and presence of outliers in hydrologic variables such as rainfall and stream flow. Outliers are values which are very large or very small compared to the rest of the data (Anscombe 1960) and which do not follow the general trend. To lessen the uncertainty that outliers cause in the estimated design events required in water engineering studies and projects, an additional mathematical tool is needed. The US Federal Agencies recommend Bulletin 17B, which provides a recommended procedure for the treatment of outliers, including outlier deletion (McCuen 2004; Griffis and Stedinger 2007). However, removing outliers from hydrologic data could lead to underestimation of design hydrologic quantities, while the use of hydrologic data that contains known outliers could lead to overestimation of these quantities. Neither of these options is economically feasible.

Many researchers have attempted to address extreme hydrologic variables using alternative methods to conventional frequency analysis. Yue (2000, 2001), Zhang and Singh (2006) and Lee et al. (2010a) applied a bivariate distribution to address joint probabilistic behavior. Strupczewski et al. (2001), Katz et al. (2002), Cunderlik and Burn (2003), Khaliq et al. (2006), Park et al. (2011), and

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Seo et al. (2012) proposed various methods for taking non-stationary hydrologic observations into consideration.

One of the main issues in hydrologic frequency analysis is the existence of mixed distributions, or multiple populations (Hirschboeck 1987). In South Korea, there are two different peaks in the annual distribution of daily rainfall. The first peak occurs in July, due to stationary convective fronts such as the Changma. The second peak occurs in August and is due to tropical storms such as typhoons (Lee et al. 2010b). Typhoons and the Changma contribute about 25 and 40 %, respectively, to the annual rainfall in the major river basins (Kim and Jain 2011). Extreme rainfall in South Korea is the result of typhoon-induced storms and convective storms during the monsoon season.

Rizwan and Kim (2013) investigated empirically the effects of outliers on rainfall depth–duration–frequency curves using a mixed Gumbel distribution, and proposed that the use of this distribution on outlier-containing hydrologic data was a good choice for the economical and safe design of hydrologic structures. Maximum daily rainfall must be categorized as either typhoon or convective rainfall for analysis using mixed distributions. A thorough description of the mixed models used in this study and their results are discussed in the following sections. In this study, three mixed distributions (mixed Gumbel, mixed GEV and EV1-GEV) were applied to the hydrologic data outliers, and the best-fitting mixed distribution was selected.

Mixed distributions

Mixed probability distribution functions have long been used for modeling samples of data from two populations (Mood et al. 1974).

$$F(x) = pF_1(x) + (1-p)F_2(x) \tag{1}$$

In Eq. (1), p is the occurrence probability of outliers, and $F_1(x)$ and $F_2(x)$ represent the cumulative distribution functions (CDFs) for the annual maximum rainfall data with and without outliers.

Mixed Gumbel distribution

The probability of extreme rainfall occurrence was expressed using the proposed Gumbel distribution (Loaiciga and Leipnik 1999). Two continuous or discrete distributions were merged to make a mixed distribution (Yoo et al. 2005). Shimizu (1993) applied a mixed log-normal distribution to analyze rainfall data. The Gumbel distribution is also known as the Extreme Value type 1 (EV1) distribution.

The inadequacy of univariate probability distributions is surmounted by including the occurrence probability of outliers in mixed distributions. The occurrence probability of outliers cannot be included in conventional univariate probability distributions. If $F_1(x)$ and $F_2(x)$ as shown in Eq. (1) belong to a Gumbel distribution (NERC 1975), then the five-parameter cumulative distribution function of the mixed Gumbel distribution is as follows:

$$F(x) = p\exp^{-\exp^{-\left(\frac{x-\mu_1}{\delta_1}\right)}} + (1-p)\exp^{-\exp^{-\left(\frac{x-\mu_2}{\delta_2}\right)}} \tag{2}$$

where μ_1 and δ_1 are location and scale parameters of the first population, and μ_2 and δ_2 are location and scale parameters of the second population. The first population and second population are the annual maximum rainfall data with and without outliers, and p is the probability of outlier occurrence. The corresponding probability density function (PDF) of the mixed Gumbel distribution is represented by the following equation:

$$f(x) = \frac{p}{\delta_1}\exp^{-\left(\frac{x-\mu_1}{\delta_1}\right)}\exp^{-\exp^{-\left(\frac{x-\mu_1}{\delta_1}\right)}} + \frac{(1-p)}{\delta_2} \times \exp^{-\left(\frac{x-\mu_2}{\delta_2}\right)}\exp^{-\exp^{-\left(\frac{x-\mu_2}{\delta_2}\right)}} \tag{3}$$

Mixed general extreme value distribution

The general extreme value (GEV) distribution belongs to a family of continuous probability distributions. It combines the EV1, Frechet and Weibull distributions. The GEV has three parameters, location, scale and shape, and is suitable for large sample sizes, especially if the sample size is greater than 50. EV1 is more suitable for sample sizes below 50 (Cunnane 1989). If $F_1(x)$ and $F_2(x)$ of Eq. (1) belong to the GEV distribution (NERC 1975), then the seven-parameter CDF of the mixed GEV distribution is as follows:

$$F(x) = p \times \exp\left\{-\left[1 + \left(\frac{x-\omega_1}{\lambda_1}\right) \times \beta_1\right]^{\frac{-1}{\beta_1}}\right\} + (1-p) \times \exp\left\{-\left[1 + \left(\frac{x-\omega_2}{\lambda_2}\right) \times \beta_2\right]^{\frac{-1}{\beta_2}}\right\} \tag{4}$$

where ω_1, λ_1 and β_1 are the location, scale and shape parameters of the first population, and ω_2, λ_2 and β_2 are the location, scale and shape parameters of the second population. The first population and second population are the annual maximum rainfall data with and without outliers, and p is the probability of outlier occurrence. The corresponding PDF of the mixed GEV distribution is represented by the following equation:

Table 1 Statistics of selected rainfall gauging stations

Gauging station	Data collection period	Data collection years	Mean	Standard deviation	Coefficient of skewness	Coefficient of variation
Ganghwa	1973–2006	34	201.11	105.91	2.34	0.53
Gangneung	1961–2006	46	185.63	126.91	3.86	0.68
Buyeo	1973–2006	34	155.44	77.65	3.85	0.50
Pohang	1961–2006	46	142.57	85.74	3.33	0.60
Ulsan	1961–2006	46	158.76	83.42	2.31	0.53
Gwangju	1961–2006	46	146.97	61.24	1.89	0.42
Mokpo	1961–2006	46	131.62	57.24	2.74	0.435
Jangheung	1973–2006	34	183.41	93.5	2.69	0.51
Haenam	1973–2006	34	166.04	84.83	2.09	0.51
Goheung	1973–2006	34	186.34	101.10	2.44	0.54

$$f(x) = \frac{p}{\lambda_1} \times \exp\left\{ -\left[1 + \left(\frac{x-\omega_1}{\lambda_1}\right) \times \beta_1\right]^{\frac{-1}{\beta_1}} \right\} \times \left[1 + \left(\frac{x-\omega_1}{\lambda_1}\right) \times \beta_1\right]^{-1-\frac{1}{\beta_1}} + \frac{(1-p)}{\lambda_2} \times \exp\left\{ -\left[1 + \left(\frac{x-\omega_2}{\lambda_2}\right) \times \beta_2\right]^{\frac{-1}{\beta_2}} \right\} \left[1 + \left(\frac{x-\omega_2}{\lambda_2}\right) \times \beta_2\right]^{-1-\frac{1}{\beta_2}} \tag{5}$$

Mixed EV1 and GEV distribution

If $F_1(x)$ of Eq. (1) belongs to the EV1 distribution and $F_2(x)$ of Eq. (1) belongs to the GEV distribution, then the six-parameter CDF of the mixed model (EV1–GEV) is as follows:

$$F(x) = p\exp^{-\exp^{-\left(\frac{x-\mu}{\delta}\right)}} + (1-p) \times \exp\left\{ -\left[1 + \left(\frac{x-\omega}{\lambda}\right) \times \beta\right]^{\frac{-1}{\beta}} \right\} \tag{6}$$

where μ and δ are the location and scale parameters of the first population and ω, λ and β are the location, scale and shape parameters of the second population. The first population and second population are the annual maximum rainfall data with and without outliers, and p is the probability of outlier occurrence. The corresponding PDF of the mixed GEV distribution is represented by the following equation:

$$f(x) = \frac{p}{\delta_1}\exp^{-\left(\frac{x-\mu}{\delta}\right)}\exp^{-\exp^{-\left(\frac{x-\mu}{\delta}\right)}} + \frac{(1-p)}{\lambda} \times \exp\left\{ -\left[1 + \left(\frac{x-\omega}{\lambda}\right) \times \beta\right]^{\frac{-1}{\beta}} \right\} \left[1 + \left(\frac{x-\omega}{\lambda}\right) \times \beta\right]^{-1-\frac{1}{\beta}} \tag{7}$$

The relationship between the CDF or non-exceedance probability and the return period, taking into consideration the outlier occurrence probability, is shown in Eq. (8).

$$F_{\text{mixed}} = \left(1 - \frac{1}{T}\right) - (1-p). \tag{8}$$

Application and discussions

There are approximately 57 rainfall gauging stations in South Korea. The Grubbs outlier test (Grubbs 1969) was performed on the 24-h annual maximum rainfall observation from the rainfall gauging stations to check for the existence of outliers. This study used the 24-h annual maximum rainfall data from ten rainfall gauging stations in South Korea, which were chosen because they had at least one outlier. Table 1 shows the statistics (mean, standard deviation, coefficient of skewness and coefficient of variation) about the selected rainfall gauging stations along with their data collection period.

Graphical representation of the 24-h annual maximum rainfall observation from Pohang gauging station is shown

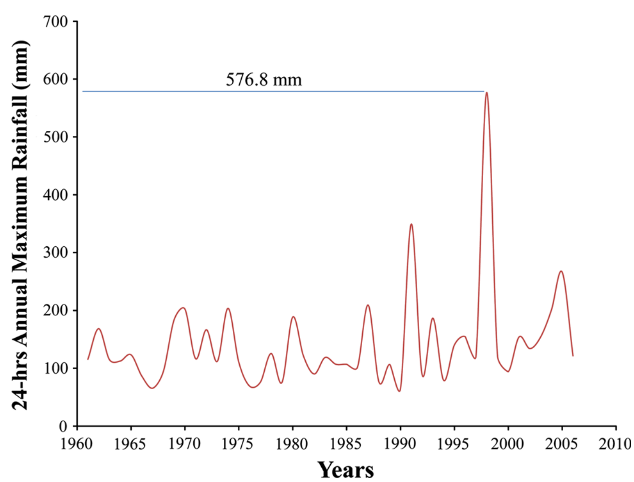

Fig. 1 24-h annual maximum rainfall observation at Pohang Gauging Station, South Korea

Table 2 Parameters of mixed distributions at selected rainfall gauging stations

gauging stations	Population	Mixed Gumbel Model		Mixed GEV model			EV1–GEV model		
		Location parameter	Scale parameter	Location parameter	Scale parameter	Shape parameter	Location parameter	Scale parameter	Shape parameter
Ganghwa	1st	153.45	82.579	149.84	50.615	0.30981	153.45	82.579	–
	2nd	153.91	59.681	149.98	49.23	0.17142	149.98	49.23	0.17142
Gangneung	1st	128.52	98.953	131.95	54.664	0.29413	128.52	98.953	–
	2nd	137.83	55.945	135.47	56.214	0.03812	135.47	56.214	0.03812
Buyeo	1st	120.5	60.542	122.9	27.643	0.38243	120.5	60.542	–
	2nd	126.37	30.303	124.9	28.707	0.07787	124.9	28.707	0.07787
Pohang	1st	103.98	66.854	104.84	35.598	0.33215	103.98	66.854	–
	2nd	107.71	43.68	106.06	35.894	0.14893	106.06	35.894	0.14893
Ulsan	1st	121.22	65.045	119.6	41.088	0.2789	121.22	65.045	–
	2nd	121.03	41.57	120.02	39.416	0.0549	120.02	39.416	0.0549
Gwangju	1st	119.41	47.75	117.92	36.566	0.18203	119.41	47.75	–
	2nd	118.98	39.501	118.02	35.815	0.0806	118.02	35.815	0.0806
Mokpo	1st	105.85	44.629	105.59	30.236	0.22541	105.85	44.629	–
	2nd	107.59	31.004	106.64	30.324	0.04296	106.64	30.324	0.04296
Jangheung	1st	141.33	72.905	139.88	43.516	0.30344	141.33	72.905	–
	2nd	143.47	48.066	141.02	43.255	0.10966	141.02	43.255	0.10966
Haenam	1st	127.87	66.14	124.44	45.029	0.26277	127.87	66.14	–
	2nd	127.94	48.641	124.64	43.882	0.12311	124.64	43.882	0.12311
Goheung	1st	140.84	78.83	138.71	50.15	0.27704	140.84	78.83	–
	2nd	141.68	55.859	139.52	49.383	0.10848	139.52	49.383	0.10848

in Fig. 1. The Pohang rainfall gauging station is located in the middle of Korea, on the eastern side, and has rainfall observation data from 1961 to 2006. The highest rainfall, 576.8 mm, occurred in 1998 as shown in Fig. 1. The Grubbs outlier test shows that the outlier threshold for this station is 420.8 mm. Any values higher than this threshold are outliers, which means that the rainfall for 1998 is an outlier.

The average rainfall at the Pohang rainfall gauging station was 142.57 mm. Over its 46 years of operation, there are 31 years which have below-average rainfall, and 15 years with above-average rainfall. The long-term trend in rainfall is shown in Table 4 for return periods between 5 and 200 years. Other studies have shown that intense storm events in South Korea could follow a Gumbel distribution (Kwon et al. 2008; Heo et al. 2006). Mathematically, a Gumbel distribution is better than other extreme probability distributions, such as the Gamma, GEV and log-normal distributions (Loaiciga and Leipnik 1999). This study utilized three mixed distributions (mixed Gumbel, mixed GEV and EV1–GEV) to include the probability of outliers occurring. The parameters for the three mixed distributions were estimated by the method of moments. Their values are shown in Table 2.

Table 3 Calculated SE (mm) for selected rainfall gauging stations

Gauging station	Mixed Gumbel model	Mixed GEV model	EV1–GEV model	Best model
Ganghwa	2.931	2.958	2.946	Mixed Gumbel
Gangneung	4.430	4.474	4.453	Mixed Gumbel
Buyeo	2.411	2.425	2.424	Mixed Gumbel
Pohang	2.622	2.648	2.636	Mixed Gumbel
Ulsan	2.000	2.016	2.010	Mixed Gumbel
Gwangju	1.533	1.548	1.541	Mixed Gumbel
Mokpo	1.654	1.672	1.663	Mixed Gumbel
Jangheung	2.710	2.735	2.724	Mixed Gumbel
Haenam	2.196	2.217	2.208	Mixed Gumbel
Goheung	2.771	2.797	2.786	Mixed Gumbel

Standard error of fit

Kite (1988) defined the minimum standard error of fit (SE) criterion for the selection of the best-mixed distribution for each station. SE is calculated using Eq. (9). The standard error values calculated by Eq. (9) for the mixed Gumbel distribution were compared with the values produced by the mixed GEV and EV1–GEV distributions.

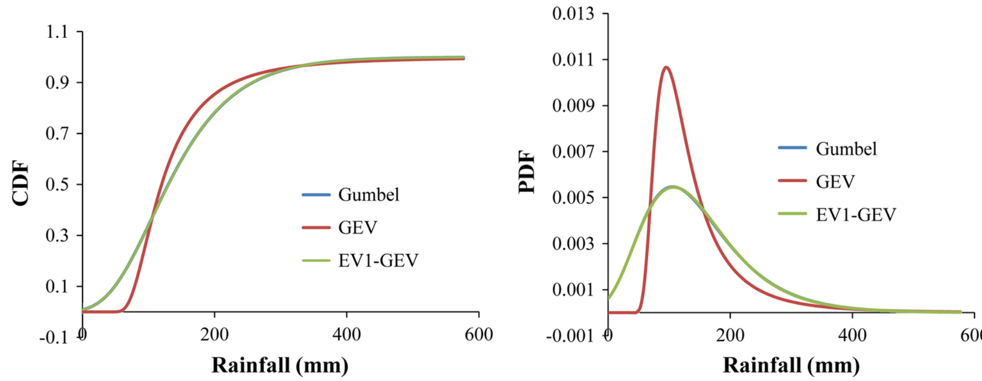

Fig. 2 CDF and PDF of the mixed distributions at Pohang Gauging Station

Table 4 Design rainfall (mm) for the best-fitted distribution (mixed Gumbel) at selected rainfall gauging stations

Return period (years)	Rainfall gauging stations									
	Ganghwa	Gangneung	Buyeo	Pohang	Ulsan	Gwangju	Mokpo	Jangheung	Haenam	Goheung
5	267.07	272.81	208.41	198.85	212.46	185.97	170.45	244.44	220.07	249.59
10	319.44	342.39	249.62	243.30	255.16	217.39	200.98	292.02	262.66	299.69
20	358.24	401.73	282.48	279.66	290.02	242.26	226.44	325.50	296.25	337.24
50	402.78	463.26	314.20	318.36	327.30	270.51	252.76	367.68	330.45	379.35
100	421.93	494.25	328.82	337.12	345.40	283.82	265.74	385.05	346.03	397.66
200	433.36	513.68	337.47	348.83	356.69	292.10	273.83	395.46	355.37	408.63

$$SE = \left[\sum_{i=1}^{n} (g_i - h_i)^2 \bigg/ (n - q) \right]^{1/2} \tag{9}$$

where g_i, $i = 1 \ldots n$, are the probabilities of the recorded events, h_i, $i = 1 \ldots n$, are the magnitudes of the recorded events from the probability distribution, n is the length of the recorded event and q is the number of estimated parameter of the mixed distribution.

The values of q for the mixed Gumbel distribution, mixed GEV distribution and Evi–GEV model were 5, 7 and 6, respectively.

The values of SE for each gauging station and each mixed distribution, and the best model or distribution on the basis of minimum SE for each gauging station are shown in Table 3.

The mixed Gumbel distribution was found to be a good fit for the rainfall data on the basis of its low SE value. For each rainfall gauging station used in this study, the mixed Gumbel distribution had the lowest standard error. Figure 2 shows the PDF and CDF of the Pohang rainfall gauging station. The PDF for the mixed Gumbel, mixed GEV and EV1–GEV distributions were calculated using Eqs. (3), (5) and (7), respectively. The CDF for the mixed Gumbel, mixed GEV and EV1–GEV distributions were calculated using Eqs. (2), (4) and (6), respectively.

The CDF and PDF for the mixed Gumbel and EV1–GEV distributions, which were very similar, are shown by the blue and green lines in Fig. 2, and the CDF and PDF for the mixed GEV distribution is shown by the red line.

The final design rainfall values (mm) calculated using the mixed Gumbel distribution for various return periods (years) at each of the selected rainfall gauging station are shown in Table 4.

A comparison of computed design rainfall values (measured in mm) for various return periods (measured in years) for the three mixed distributions at each of the selected rainfall gauging stations are shown in Table 5.

The computed design rainfall values for the mixed Gumbel distribution and the EV1–GEV distribution for various return periods are very similar, as shown in Table 5, and the design rainfall values calculated using the mixed GEV distribution are smaller than those from the mixed Gumbel or EV1–GEV distributions.

Conclusions

Ten rainfall gauging stations across South Korea with at least one outlier in their 24-h annual maximum rainfall data were selected for the study. As the commonly used univariate probability distribution is unsuitable for use on data with outliers, three mixed distributions (mixed Gumbel, mixed GEV, EV1–GEV) were used. The mixed Gumbel

Table 5 Comparison of design rainfall (mm) for selected rainfall gauging stations

Gauging stations	Distribution	Return period (years)					
		5	10	20	50	100	200
Ganghwa	Mixed Gumbel	267.07	319.44	358.24	402.78	421.93	433.36
	Mixed GEV	237.54	291.83	339.38	402.10	432.52	451.70
	EV1–GEV	266.49	318.76	358.24	402.50	421.88	433.45
Gangneung	Mixed Gumbel	272.81	342.39	401.73	463.26	494.25	513.68
	Mixed GEV	232.02	299.01	366.02	450.45	495.77	525.39
	EV1–GEV	272.68	342.07	392.80	462.71	493.71	513.21
Buyeo	Mixed Gumbel	208.41	249.62	282.48	314.20	328.82	337.47
	Mixed GEV	168.43	201.12	233.59	272.49	290.96	302.25
	EV1–GEV	208.46	249.41	277.05	313.50	328.10	336.77
Pohang	Mixed Gumbel	198.85	243.30	279.66	318.36	337.12	348.83
	Mixed GEV	170.61	213.60	256.36	309.69	339.50	359.47
	EV1–GEV	198.59	242.81	279.66	317.89	336.77	348.58
Ulsan	Mixed Gumbel	212.46	255.16	290.02	327.30	345.40	356.69
	Mixed GEV	190.55	234.21	275.99	327.58	355.67	374.30
	EV1–GEV	212.35	255.07	290.02	327.37	345.54	356.88
Gwangju	Mixed Gumbel	185.97	217.39	242.26	270.51	283.82	292.10
	Mixed GEV	176.06	208.93	238.42	274.06	292.49	304.49
	EV1–GEV	185.86	217.29	242.26	270.55	283.91	292.23
Mokpo	Mixed Gumbel	170.45	200.98	226.44	252.76	265.74	273.83
	Mixed GEV	158.21	189.85	218.21	254.34	272.72	284.65
	EV1–GEV	170.41	200.87	226.44	252.59	265.60	273.73
Jangheung	Mixed Gumbel	244.44	292.02	325.50	367.68	385.05	395.46
	Mixed GEV	217.80	266.51	307.75	363.30	389.29	405.61
	EV1–GEV	244.26	291.47	325.50	366.86	384.31	394.79
Haenam	Mixed Gumbel	220.07	262.66	296.25	330.45	346.03	355.37
	Mixed GEV	201.02	246.64	286.21	335.55	359.27	374.09
	EV1–GEV	219.75	262.15	296.25	330.09	345.80	355.23
Goheung	Mixed Gumbel	249.59	299.69	337.24	379.35	397.66	408.63
	Mixed GEV	223.93	275.16	319.18	376.14	403.29	420.37
	EV1–GEV	249.25	299.23	337.24	379.10	397.52	408.58

distribution produced the smallest SE at all stations. As such, the mixed Gumbel distribution is the best-mixed distribution to fit to 24-h annual maximum rainfall data with outliers.

The differences between design rainfall values calculated for large return periods can be considerable, especially if an unsuitable distribution is chosen. Hydraulic structures and projects that use inaccurate design rainfall values might become unsafe and economically unfeasible. The design rainfall values computed using the best-fitted mixed distribution proposed in this paper could be a good option for creating economically feasible and safe hydrologic systems.

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