Field Assessment of Friction Head Loss and Friction Correction Factor Equations

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Abstract: Friction head loss equations and friction correction factors were evaluated and compared to field observations collected from thirty center pivots with laterals made of PVCs. The friction head loss equations include Darcy-Weisbach (D-W), Hazen-Williams (H-W), and Scobey, in addition to a proposed equation valid for smooth and rough pipe types and for all turbulent flow types. The proposed equation was developed by combining the equations of D-W and H-W, along with the multiple nonlinear regression technique. The friction correction factors were computed by using the typical Christiansen, modified Christiansen, Anwar, and Alazba formulae. The evaluation has been based on statistical error techniques with observed values as a reference. With the combination of modified Christiansen, Anwar, and Alazba formulae, the results revealed that the magnitudes of friction head loss calculated by using the D-W, H-W, and proposed equations were in agreement with field observations. The root mean square deviation (RMSD) values ranged from 1.6 to 1.7 m. As expected, and when the typical Christiansen friction correction factor was used with the D-W, H-W, and proposed equations, the results showed poor agreement between observed and computed friction head loss values. This was clearly reflected by the high RMSD values that ranged from 5.4 to 5.9 m. On the other hand, agreement occurred between observed friction head loss values and those calculated by using the Scobey equation, invalid for PVC pipe type, when combined with the typical Christiansen formula. This interesting finding led to improved results of the Scobey equation through a developed C_s coefficient suitably valid for PVC pipe type through analytically mathematical derivation; accordingly, the RMSD value dropped from approximately 8.6 to 1.6 m. **DOI: 10.1061/(ASCE)IR.1943-4774.0000387.**

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Introduction

Center-pivot irrigation machines are among the most popular systems for irrigating general field crops and are widely used on sprinkler-irrigated lands. The center pivots are best suited for regions where vast farmland and sufficient water resources, coupled with limited manpower, are available. They are often preferred by farmers because of their robustness and automation possibilities (Dechmi et al. 2003). A center-pivot system consists of a sprinkler line rotating in a circle around a base pipe structure in the center of the field (single pivot point) so that the irrigated section takes a circular shape including parts of circles less than 360°. A single center pivot can cover 80–90% of the area of a squared field. The pivot base structure is the source of water, power, and control wires. Center pivots can operate on widely variable terrain with slopes as much as 30% with proper design, although an upper limit of 15% slope is generally recommended. The water is introduced at the pivot point and flows outward through the pivot laterally supplying each individual sprinkler head.

The selection of pipe diameter has vital influence on the capital cost of the whole center-pivot system because it is interrelated to head loss and, accordingly, to the selection of pumps and motors. Therefore, the determination of friction head loss is a key element toward the selection of an appropriate pipe diameter and an essential parameter in the design of center-pivot irrigation systems. Friction head loss calculation, in particular, is a challenging task for hydraulic engineers because of many involved variables. In center pivots, the flow variability in the main pipe, the number and type of openings along the pipe, and the radial movement of the system are some of the complexities that have to be resolved.

Irrigation engineers have traditionally relied on physical and empirical equations for estimating and evaluating energy head loss attributed to friction (h_f) in center-pivot laterals. The Darcy-Weisbach (D-W) equation is fundamentally sounder than other empirical and approximate equations (Kamand 1988). The D-W equation is given as follows:

$$h_f = \frac{K \cdot f \cdot L}{D^5} \cdot Q^2 \tag{1}$$

where h_f = friction head loss where there is only one outlet (m); K = units conversion factor equal to 0.0826; f = D-W friction factor; L = pipe length (m); Q = flow rate (m^3/s); and D = inside pipe diameter (m).

Churchill (1977) introduced a friction factor equation that is valid for both rough and smooth pipes and for the full range of laminar, transition, and fully turbulent flow regimes of the Moody diagram. The Churchill equation is written as follows:

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$$f = 8\left[\left(\frac{8}{\mathsf{R}}\right)^{12} + \frac{1}{(A+B)^{1.5}}\right]^{\frac{1}{12}} \tag{2}$$

where R = dimensionless Reynolds number equal to VD/υ; $V =$ = flow velocity in m/s and $\upsilon =$ = kinematic viscosity in m^2/s; and A and $B =$ = variables computed as mentioned in Eqs. (3) and (4), respectively

$$A = \left\{-2.457\,\mathrm{Ln}\left[\left(\frac{7}{\mathsf{R}}\right)^{0.9} + 0.27\left(\frac{e}{D}\right)\right]\right\}^{16} \tag{3}$$

$$B = \left(\frac{37530}{\mathsf{R}}\right)^{16} \tag{4}$$

where e = equivalent roughness height (mm).

An empirical formula equation known as Hazen-Williams (H-W) equation is commonly used in irrigation hydraulics to calculate friction head loss. The H-W equation has the following form (Williams and Hazen 1933):

$$h_f = \frac{K \cdot L}{C_{\mathrm{HW}}^{1.852} \cdot D^{4.871}} \cdot Q^{1.852} \tag{5}$$

where K = empirical factor equal to 1.22×10^{10} for L in m, D in mm, and Q in l/s; and C_{HW} = H-W coefficient that depends on the flow media.

Other common empirical equations exist to estimate the head loss as reported by Scobey (1930) and Watters and Keller (1978). The Scobey equation is given as follows:

$$h_f = \frac{K \cdot C_s \cdot L}{D^{4.9}} \cdot Q^{1.9} \tag{6}$$

where K = empirical factor equal to 2.05×10^{12} for L in m, D in mm, and Q in m^3/s; and C_s = Scobey coefficient of retardation. The C_s equal to 0.43, 0.40, and 0.39 for portable aluminum pipe with couplers for 6, 9, and 12 m longitude, respectively.

The head loss caused by friction in a pipeline with multiple outlets along its longitude will be less than that without outlets because of decreasing flow capacity along the length of the pipeline. When computing the friction head loss in the lateral line, it is logical to start computing the discharge and friction loss between two successive outlets at the last outlet on the line and work backward to the supply line i.e., the stepwise analysis. This tedious process has been simplified for nonrotating sprinkler laterals by multiplying the friction head loss calculated for a pipe with one outlet by a friction correction factor, first introduced by Christiansen (Christiansen 1942). The typical Christiansen friction correction factor valid for outlets with relatively constant rates is written as follows:

$$F = \frac{1}{m+1} + \frac{1}{2N} + \frac{\sqrt{(m-1)}}{6N^2} \tag{7}$$

where F = friction correction factor for flow decreasing linearly; m = velocity exponent in the formula used for computing head loss caused by friction; and N = number of outlets along the pipeline.

When the distance from the pipe inlet to the first outlet is half of the outlet spacing, F should be computed as follows:

$$F_{0.5} = \frac{2NF - 1}{2N - 1} \tag{8}$$

The friction correction factor (F), derived by Christiansen (1942), is valid under the assumption that the flow rates from sprinklers evenly spaced along a lateral are approximately identical. This assumption holds true for linear-move sprinkler systems when the variation of flow rate along a lateral is minimal. However, this is not the case with center pivots where the desired flow rates from the sprinklers located on the lateral pipe vary increasingly and considerably as water runs away from the pivot. This is because, for any rotation of the center pivot, the outer end of the lateral must irrigate a greater area than the inner (pivot) end of the lateral (Anwar 1999). Under these conditions, the use of the "F" factor derived by Christiansen (1942) is inapplicable, and an alternative friction correction factor must be used in sizing the lateral of a center-pivot irrigation system.

Reddy and Apolayo (1988) developed a friction correction factor for center pivots for a finite number of outlets and is calculated as follows:

$$F' = \frac{1}{N}\left[1 + \sum_{i=2}^{N}\left(1 - \frac{2}{N^2}\sum_{j=1}^{i-1} j\right)^m\right] \tag{9}$$

where F' = friction correction factor for flow decreasing nonlinearly (center pivots) with N outlets; i = integer $(2, 3, 4, \ldots, N)$; and j = integer $(1, \ldots, i)$.

Anwar (1999) proposed two formulas that are valid for any number of outlets, $N \geq 1$. For constant spacing (increasing varied outflow), the formula is written as follows:

$$F' = \frac{1}{N^{2m+1}}\sum_{i=1}^{N}(2Ni - i^2)^m \tag{10}$$

For constant outlet discharge (decreasing varied space), the formula can be written as follows:

$$F' = \frac{1}{N^{m+0.5}}\sum_{i=1}^{N} i^m\left(\sqrt{N-i+1} - \sqrt{N-i}\right) \tag{11}$$

Alazba (2005) developed a friction correction factor formula valid for center-pivots lateral and is of the following exponentially simple form:

$$F' = \frac{1 + \frac{1}{N}}{e^{\frac{m}{\pi}}} \tag{12}$$

Alazba (2005) further modified the Christiansen formula to be valid for center pivots, and the modified equation takes the following formulation:

$$F' = \left[\frac{1}{m+1} + \frac{1}{2N} + \frac{\sqrt{m-1}}{6N^2}\right]^{0.567} \tag{13}$$

where all terms have previously been identified.

System design, specifications, and installations are readily imported from popular manufacturers all over the world and operated by local workers and engineers. Variability between parameters, such as head loss, obtained under ideal and actual operating conditions put the center-pivot systems in low efficiency, thus, affecting the economical return of the agricultural enterprise. Thus, system evaluation under field operating conditions is necessary to maximize the system efficiency and economic return. One of the most important parameters that needs checking and optimization is the friction head loss calculation.

It is clear from the previous sections that several equations are used to calculate the friction head loss of a pipe with multiple outlets where discharge non-linearly varies as in the case of center-pivots systems. Engineers may encounter some difficulties in calculating these various types of friction head losses and face ambiguity in selecting the appropriate equation. The design of irrigation systems requires a thorough knowledge of the previously mentioned equations with regard to their accuracy and ease of application. Therefore, the objectives of this study were to evaluate and examine the appropriate equations for friction head loss calculation on real field operating center-pivot systems, and to develop a simple equation that predicts friction head loss accurately.

Materials and Methods

Experimental Site

Field observations were made at 30 center pivots in a farm located in Riyadh, Saudi Arabia, with 20°65′ latitude and 45°15′ longitude. The pivots were of two laterals having lengths of 395 and 417 m, and 152 mm of diameter. These laterals are made of PVC in a new situation with the absolute value of roughness equal to 0.015. The number of outlets (N) ranged from 156–204. Each tower is moved by an electric pump. Analog pressure gauges suitably located at inlet and outlet ends of the pivot lateral were used to measure the pressures at both ends and, hence, to calculate the friction head losses. An ultrasonic device was installed on the pivot lateral to measure the inflow rate (Q) by installing two bands around the outer perimeter of pipeline. Therefore, flow velocity (V) and Reynolds number (R) were calculated. Table 1 summarizes the values of several parameters for the 30 center pivots.

Friction Head Loss Equations

In addition to the proposed equation, three well-known equations; namely, D-W [Eq. (1)], H-W [Eq. (5)], and Scobey [Eq. (6)] were used to calculate the friction head losses in center-pivots laterals operating at a real field level.

The proposed equation stems from the fact that C_{HW} in the H-W equation cannot be considered constant with pipe diameter (D) [Allen (1996) and Valiantzas (2005)] and, subsequently, leads to significant errors in computing h_f. Although the D-W equation results in accurate h_f, it is, however,quite tedious to determine h_f in frequently routine applications. Thus, it was intended to develop a simpler, yet accurate, equation as compared to the D-W equation.

The proposed equation was obtained by combining the D-W and H-W equations, along with mathematical simulation and manipulation and regression analysis. The developed equation can be written as follows:

$$h_f = \frac{K \cdot L}{C_{\mathrm{DW}} \cdot D^{5.2}} \cdot Q^2 \qquad (14)$$

where K = empirical factor equal to 0.0826 for L in m, D in m, and Q in $\mathrm{m^3/s}$; and C_{DW} = numerical friction coefficient.

In Eq. (14), the C_{DW} coefficient is a key parameter and substantially determines the accuracy of the h_f calculation. Exact C_{DW} consideration implies that it is taken as a function of the V, D, and e variables and leads to an exact h_f determination, i.e., identical to h_f computed from the D-W equation. On the other hand, approximate h_f determination results when C_{DW} is taken as a function of either D and e, or e only. A function only of e gives the value of h_f that closely matches with the h_f value computed by using the H-W equation.

For an exact h_f solution, C_{DW} can simply be computed by using the following equation:

$$C_{\mathrm{DW}} = \frac{1}{fD^{0.2}} \qquad (15)$$

Eq. (15) was used to simulate C_{DW} for a wide range of the input variables, which include V, D, and e. The kinematic viscosity (υ) was considered constant with a value equal to $1.006 \times 10^{-6}\ \mathrm{m^2/s}$. For an approximate solution of h_f when C_{DW} is considered as a function of V and D variables, an empirical C_{DW} equation was ultimately approached through a multiple nonlinear regression technique. With appropriate increments and covering typical ranges of variation, the V varied between 0.5 and 10 m/s, the D ranged from 10–1,000 mm, and the e was taken between 0 and 10 mm. Therefore, the obtained C_{DW} values, Eq. (16), imply that Eq. (14) is valid for both rough and smooth type pipes and for the full range of turbulent flows ($\mathsf{R} > 4{,}000$).

From the analysis (not shown), it was found that a good correlation exists between the V and both the D and the e. Therefore, averaged C_{DW} values were considered over the velocity ranges. In fact, an average V value of 2.65 m/s was found to be representative of all other V values and can be used with insignificant errors. Nevertheless, it was found that the magnitude of C_{DW} has a constant value of 110 for e ranging between 0.00 and 0.01. On the other hand, for e values ranging between 0.01 and 10, C_{DW} was found to be accurately calculated by using the following equation:

$$\begin{aligned} &C_{\mathrm{DW}} = a - b\ln e \\ &\text{where } a \text{ is computed from} \quad a = \frac{15.089 + 1.593D}{1 + 0.03D} \\ &\text{and } b \text{ is computed from} \quad b = \frac{17.96 + 0.12D}{1 + 0.011D} \end{aligned} \qquad (16)$$

Table 1. Average Values of Parameters

Items	Maximum	Minimum	Average
Lateral length (L) = 395 m			
Inflow rate (Q), $\mathrm{m^3/h}$	313.7	195.4	249.1
Outlets number (N)	186	156	172
Flow velocity (V), m/s	4.78	2.98	3.79
Friction head loss (h_f), m	21.65	10.43	15.07
Reynolds number (R)	7.28×10^5	4.54×10^5	5.78×10^5
Lateral length (L) = 417 m			
Inflow rate (Q), $\mathrm{m^3/h}$	303.3	199.9	247.2
Outlets number (N)	204	162	176
Flow velocity (V), m/s	4.62	3.05	3.77
Friction head loss (h_f), m	22.06	8.62	14.50
Reynolds number (R)	7.04×10^5	4.64×10^5	5.74×10^5

Note: Average of 30 center pivots.

Friction Correction Factor Formulas

As might previously be noticed, four formulas were used to calculate the friction correction factor and subsequently the friction head losses and finally have them compared with field observations. They included the typical Christiansen equation [Eq. (7)], the modified Christiansen equation [Eq. (13)], Anwar equation [Eq. (10)], and Alazba equation [Eq. (12)]. The aim of applying the typical Christiansen equation, Eq. (7), was to emphasize its invalidity and implacability under the hydraulic conditions of center-pivots laterals.

Statistical Parameters

The agreement between measured and simulated friction head loss values was quantified with five statistical measures that included root mean square deviation (RMSD) [Eq. (17)], normalized root mean squared deviation (NRMSD) [Eq. (18)], model efficiency (ME) [Eq. (19)], overall index of model performance (OI) [Eq. (20)] and coefficient of residual mass (CRM) [Eq. (21)]. Mathematically, they are expressed as follows:

$$\mathrm{RMSD} = \sqrt{\frac{\sum_{i=1}^{n}(x_{o,i} - x_{c,i})^2}{n}} \tag{17}$$

$$\mathrm{NRMSD} = \frac{\mathrm{RMSD}}{x_{\max} - x_{\min}} \tag{18}$$

$$\mathrm{ME} = 1 - \frac{\sum_{i=1}^{n}(x_{o,i} - x_{c,i})^2}{\sum_{i=1}^{n}(x_{o,i} - \bar{x}_o)^2} \tag{19}$$

$$\mathrm{OI} = \frac{1}{2}(1 - \mathrm{NRMSD} + \mathrm{ME}) \tag{20}$$

$$\mathrm{CRM} = \frac{\left(\sum_{i=1}^{n} x_{c,i} - \sum_{i=1}^{n} x_{o,i}\right)}{\sum_{i=1}^{n} x_{o,i}} \tag{21}$$

where $x_{o,I}$ = observed value; $x_{c,i}$ = calculated value; n = number of observations; $x_{\max}$ = maximum observed value; $x_{\min}$ = minimum observed value; and $\bar{x}_o$ = averaged observed values.

RMSD has been used by different authors to compare predicted and measured parameters (Arbat et al. 2008). RMSD has the advantage of expressing the error in the same units as the variable, thus, providing more information about the efficiency of the model (Legates and McCabe 1999). The lower the RMSD or NRMSD, the more accurate the simulation is. A model efficiency (ME) value of 1.0 means a perfect fit between measured and predicted data, and this value can be negative. The OI combines the NRMSD and ME indicators to verify the performance of mathematical models. An OI value of one for a model means a perfect fit between the observed and predicted data. The CRM parameter shows the difference between observed and predicted data relative to the observe data. A zero value indicates a perfect fit, whereas, positive and negative values indicate an over-and underprediction by the model, respectively.

The friction head loss is referred to that of a pipe with no outlets and that of a pipe with outlets having constant and variable flow rates. The three cases are differentiated by the abbreviations h_f, h_f', and h_f'', respectively.

Results and Discussion

D-W Equation versus Observations

Fig. 1 depicts the relationship between measured and observed values of friction head loss h_f'' by using the D-W equation. Fig. 1 encompasses four graphs representing the four formulas used to compute F'. It is clear from the figure that the calculated h_f'' values

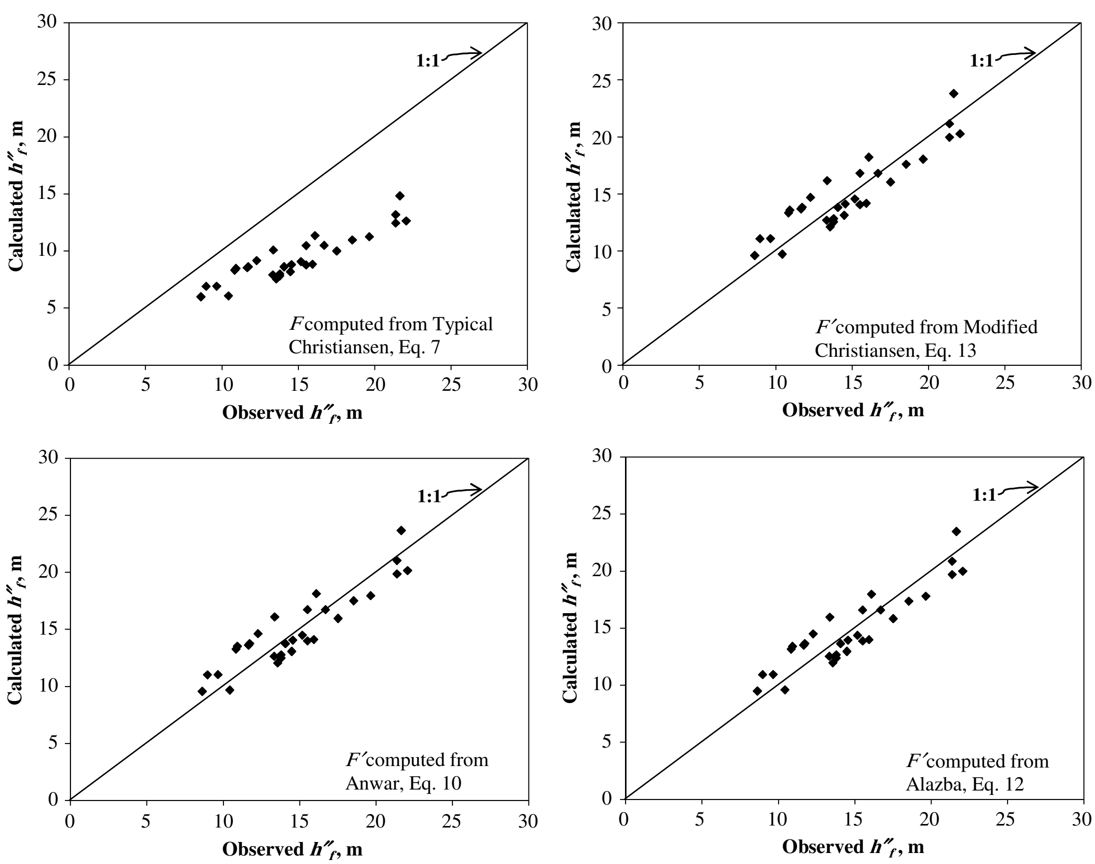

Fig. 1. Calculated versus observed friction head loss by using the D-W equation

Table 2. Statistical Parameters of Friction Head Loss for Center Pivots

Friction head loss equations	Statistical parameters	Friction correction factor formulas			
		Typical Christiansen	Modified Christiansen	Anwar	Alazba
Darcy-Weisbach	RMSD	5.80	1.61	1.61	1.60
	NRMSD	0.43	0.12	0.12	0.12
	ME	−1.39	0.81	0.82	0.82
	OI	−0.41	0.85	0.85	0.85
	SRM	0.37	−0.02	−0.01	−0.001
Hazen-Williams	RMSD	5.44	1.67	1.65	1.70
	NRMSD	0.40	0.12	0.12	0.13
	ME	−1.11	0.80	0.81	0.79
	OI	−0.25	0.84	0.84	0.83
	SRM	0.34	−0.03	−0.03	−0.04
Scobey	RMSD	1.61	8.60	8.46	8.58
	NRMSD	0.12	0.64	0.63	0.64
	ME	0.81	−4.26	−4.09	−4.24
	OI	0.85	−1.95	−1.86	−1.94
	SRM	0.01	−0.56	−0.55	−0.56
Proposed	RMSD	5.91	1.60	1.60	1.61
	NRMSD	0.44	0.12	0.12	0.12
	ME	−1.49	0.82	0.82	0.82
	OI	−0.46	0.85	0.85	0.85
	SRM	0.38	−0.0001	0.005	0.01

by using the D-W equation, coupled with the three formulas of friction correction factor, namely, modified Christiansen, Anwar, and Alazba, are in good agreement with the measured h_f'' values. Unexpectedly, the computed h_f'' values deviated from the measured h_f'' when F' was determined by the typical Christiansen formula.

Table 2 shows the results of the statistical parameters, RMSD, NRMSD, ME, OI, and SRM, which are numerical indicators used

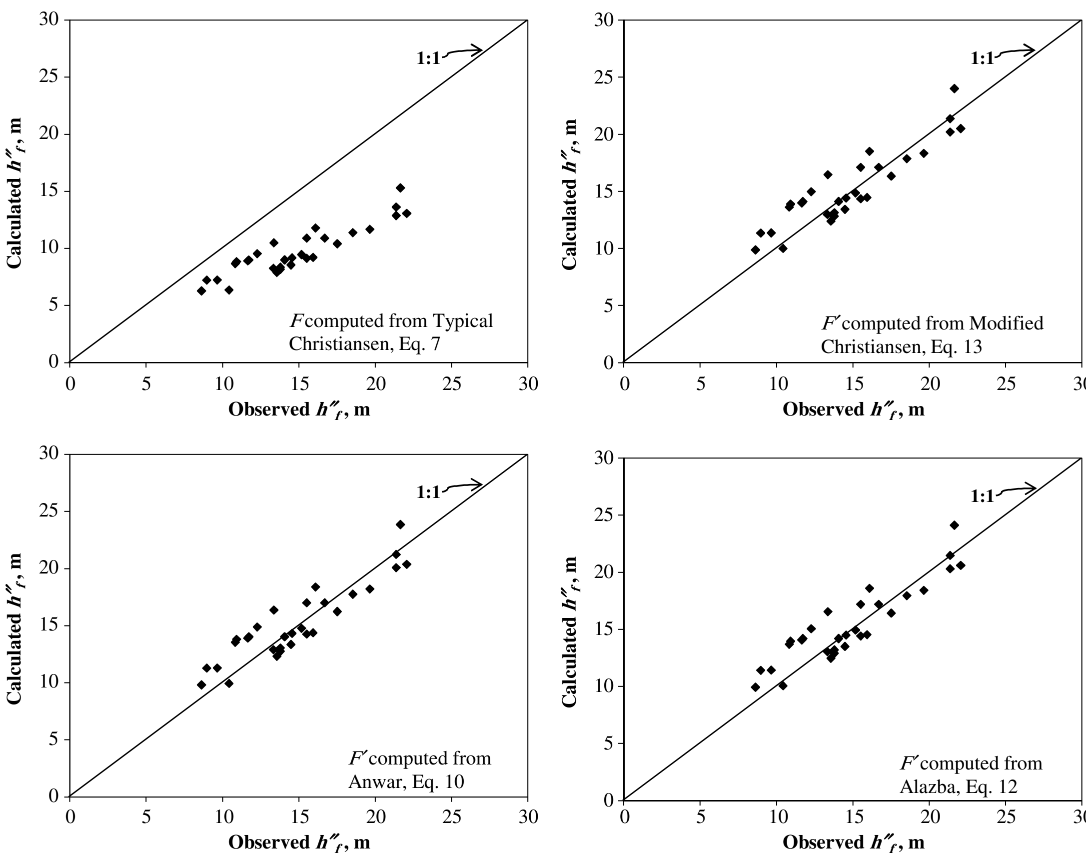

Fig. 2. Calculated versus observed friction head loss by using the H-W equation

to evaluate the agreement between measured and calculated h_f''. From Table (2), it can be realized that the values of RMSD, NRMSD, and SRM are low for F' computed from the modified Christiansen, Anwar, and Alazba formulas and high for F' computed by the typical Christiansen formula. As an instance, the RMSD equals 5.80 m for F' computed from the typical Christiansen formula and is in the range of 1.60 for the other three formulas used to determine F'. From Table 2, it can also be recognized that the statistical parameters ME and OI have high values when F' is obtained through the modified Christiansen, Anwar, and Alazba formulas indicating good agreement of measured and calculated h_f''. The values of ME and OI are, respectively, equal to -1.39 and -0.41 for F' computed from the typical Christiansen formula and fall between 0.81 and 0.85 for the three formulas used to compute F'. These findings emphasize the improper application of the typical Christiansen formula for computing $F'$and, subsequently, the determination of h_f'', which is the friction head loss of center-pivots laterals, and vice versa for the other three F' formulas.

H-W Equation versus Observations

Fig. 2 depicts the relationship between measured and observed values of friction head loss h_f'' by using the H-W equation and encompasses four graphs representing the four formulas used to compute F'. The figure clearly shows that the h_f'' values calculated by using the H-W equation, coupled with the three formulas of friction correction factor (modified Christiansen, Anwar, and Alazba), are in a good agreement with the measured h_f'' values. The computed h_f'' values, unexpectedly, deviate from the measured h_f'' when F' is determined by the typical Christiansen formula.

Table 2 indicates that the values of RMSD, NRMSD, and SRM are low for F' computed by using modified Christiansen, Anwar, and Alazba formulas and high for F' computed by the typical Christiansen formula. Although the RMSD has low values equal to 1.67, 1.65, and 1.70 m associated with the modified Christiansen, Anwar, and Alazba formulas, respectively, it has a high value equal to 5.44 m associated with the typical Christiansen formula. This, in turn, implies that the use of the later formula is inappropriate for the determination of F' and, accordingly, will lead to incorrect h_f'' calculation in the laterals of center pivots. From Table 2, it is further proved that the statistical parameters, ME and OI, have high values when F' is obtained with the modified Christiansen, Anwar, and Alazba formulas indicating good agreement of measured and calculated h_f'' values. The values of ME and OI are, respectively, equal to -1.11 and -0.25 for F' computed from the typical Christiansen formula and fall between 0.79 and 0.84 for the other three formulas used to compute F'. This in turn, emphasizes the improper use of the typical Christiansen formula for computing F' and, subsequently, the determination of h_f'' and vice versa for the other three F' formulas.

Proposed Equation versus Observations

Figs. 3–7 depict the accuracy of the derived C_{DW} form, Eq. (15), and the applicability of the proposed equation, Eq. (14), for the determination of friction head loss in a pipe. Fig. 3 shows the relative errors when C_{DW} was a constant value of 110 for the considered ranges of V, D, and e. The relative error does not exceed $\pm 9\%$ for $0 < e \leq 0.01$. Fig. 4 shows the variation of the three previously stated options of C_{DW} for values of e equal to 0.1, 1 and 10. Fig. 4 indicates that the values of the derived C_{DW}, Eq. (16), have a good agreement with exact values of C_{DW} obtained from Eq. (15). Moreover, Fig. 4 depicts that the C_{DW} cannot be considered constant with D even with high values of e. For values of e ranging between 0.01 and 10, Fig. 5 indicates that the relative error of C_{DW} values computed from Eq. (16) is within $\pm 10\%$. Tracing the error of combining Eq. (14), the proposed equation for h_f determination, and the C_{DW} equation, Eq. (16), is depicted in Fig. 6, which emphasizes that the relative error of h_f computed from Eq. (14)

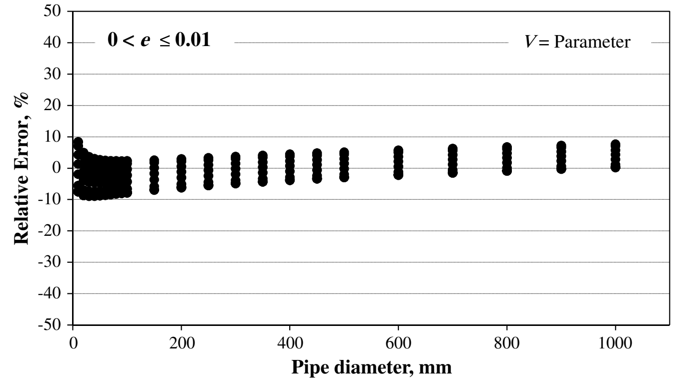

Fig. 3. Relative errors with C_{DW} taken constant and equal to 110

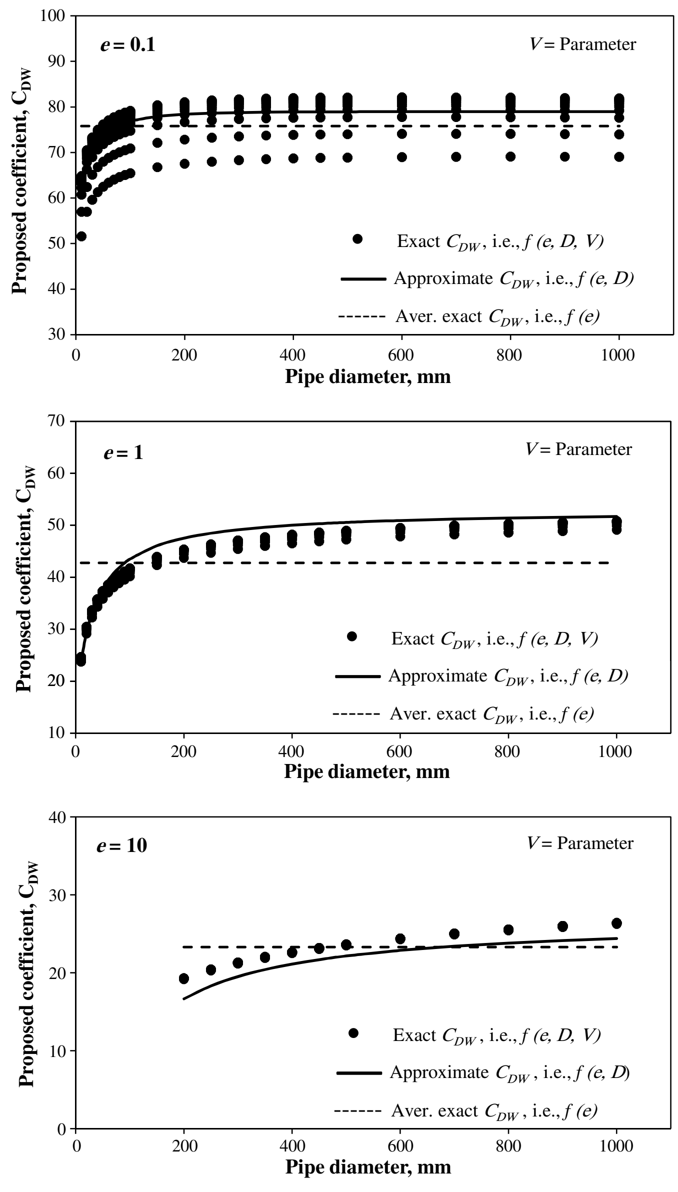

Fig. 4. Variation of C_{DW} with pipe diameter for $e = 0.1$, 1, and 10 mm

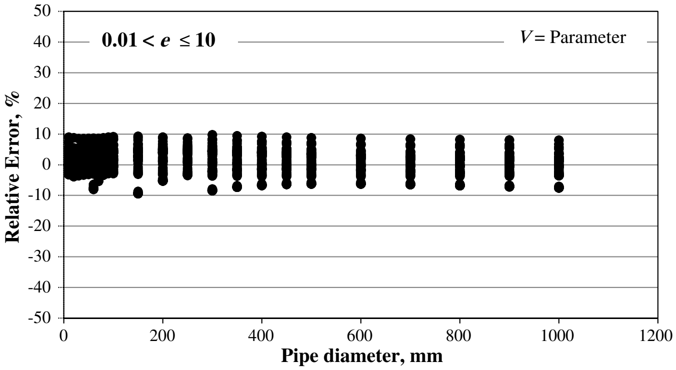

Fig. 5. Relative errors of C_{DW} calculated by Eq. (16)

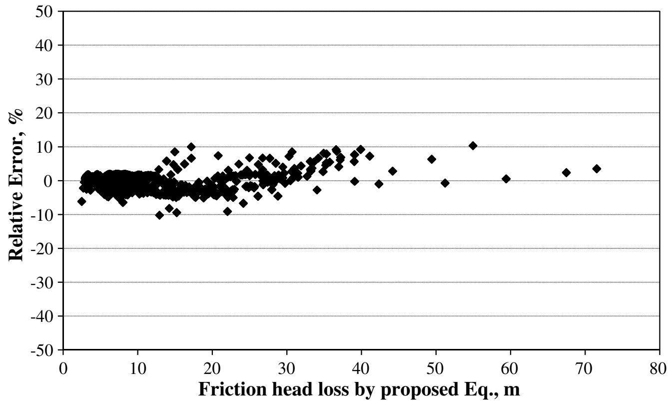

Fig. 6. Relative error of the friction head loss calculated with the proposed equation versus the D-W equation

compared to h_f computed from Eq. (1) ranges approximately from $+10$ to -10%.

Fig. 7 depicts the agreement between measured and calculated h''_f by using the proposed equation, Eq. (14), with C_{DW} computed from Eq. (16). The results shown in Fig. 7 match well with the results obtained from the D-W and H-W equations. The applicability of the proposed equation to determine the frication head loss is evident in Table 2, which shows that the magnitudes of the statistical parameters for h''_f calculated by the proposed equation do not differ from that for h''_f calculated using the D-W and H-W equations.

Scobey Equation versus Observations

Fig. 8 demonstrates the measured h''_f versus h''_f determined by the Scobey equation with C_s coefficient of 0.4 that is valid only for aluminum pipe. Fig. 8 shows a clear deviation between measured and calculated h''_f with F' computed by the modified Christiansen, Anwar, and Alazba formulas. Surprisingly, an unexpected agreement is obtained between the measured and calculated h''_f with F' computed by the typical Christiansen formula as demonstrated by the one to one curve in Fig. 8. This good agreement is obviously reflected by the values of the statistical parameters shown in Table 2, which indicates that the values of RMSD, NRMSD, ME, OI, and SRM associated with typical Christiansen formula are identical to their values of other F' formulas used with the D-W, H-W, and proposed equations. Referring to Fig. 8, this interesting result indicates the possible transformation between h'_f and h''_f, which in turn, encourages the improvement of the Scobey equation through a modification of the C_s coefficient.

The general mathematical formulation of computing the friction head loss in a pipe of multiple-outlet with flow decreasing linearly can be written as follows:

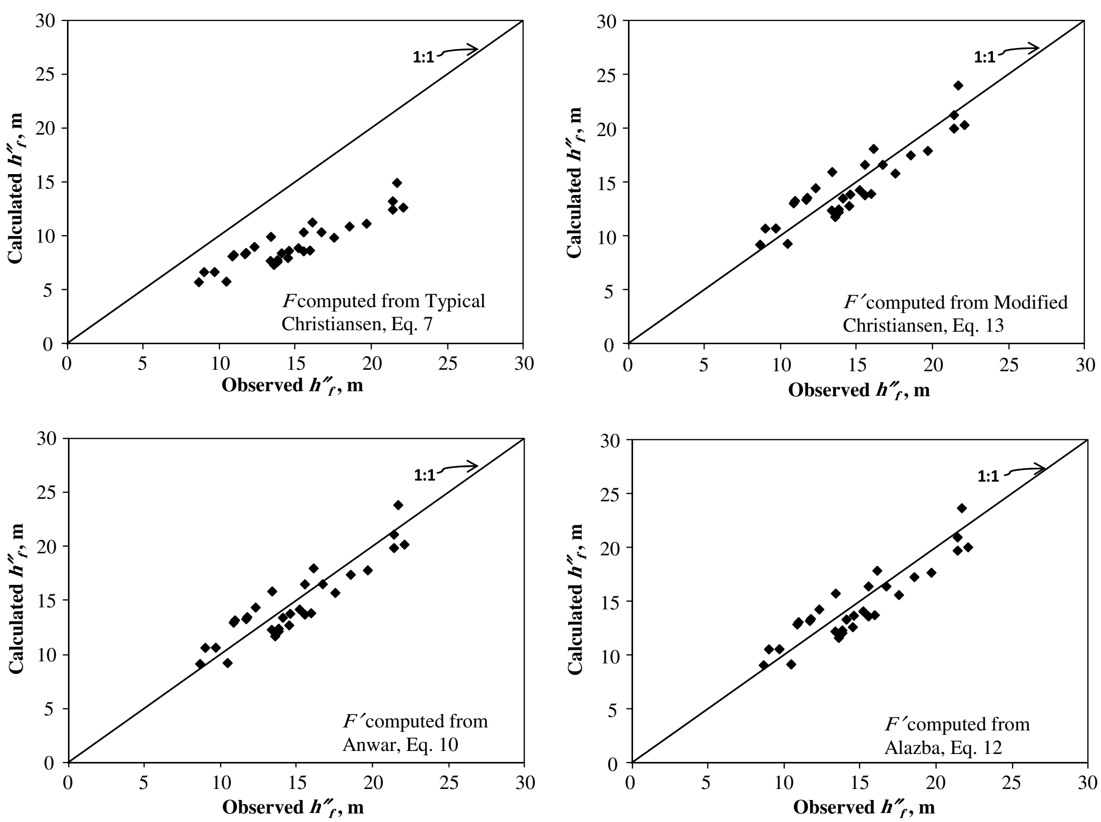

Fig. 7. Calculated versus observed friction head loss by using the proposed equation

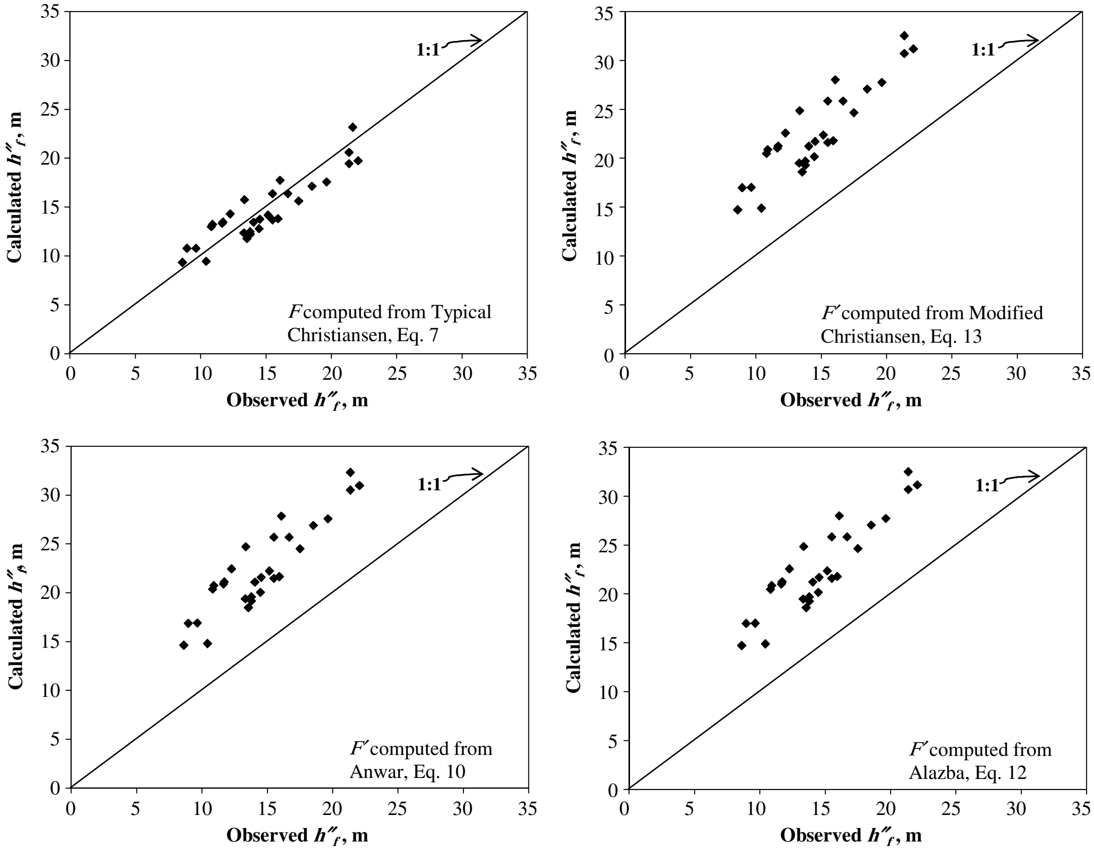

Fig. 8. Calculated versus observed friction head loss by using the Scobey equation

$$h'_f = F \cdot h_f \tag{22}$$

The formulation for a nonlinear decreased flow is

$$h''_f = F' \cdot h_f \tag{23}$$

where h'_f = energy loss attributed to friction with multiple outlets of linear decline in discharge; and h''_f = energy loss attributed to friction with multiple outlets of nonlinear decline in discharge.

It is obvious from Fig. 8 that the calculated h''_f is in agreement with the observed h''_f when F is computed by the typical Christiansen equation. This is mathematically expressed as follows:

$$h''_f = F \cdot h_f \tag{24}$$

Multiplying Eq. (24) by $\frac{F'}{F'}$, the following form is obtained:

$$h''_f = \alpha \cdot F' \cdot h_f \tag{25}$$

where

$$\alpha = \frac{F}{F'} \tag{26}$$

Recalling Eq. (13), the relationship between the typical and modified Christiansen friction correction factor can be expressed as follows:

$$F = (F')^{\frac{1}{0.567}} \tag{27}$$

For a large number of outlets, Eq. (12) can be simplified to (Alazba 2005)

$$F' = \frac{1}{e^{\frac{m}{\pi}}} \tag{28}$$

Combining Eqs. (27) and (28), the α parameter is computed by the following form:

$$\alpha = \left(\frac{1}{e^{\frac{m}{\pi}}}\right)^{\frac{1}{0.567}-1} \tag{29}$$

With the compensation of m by 1.9 as the velocity exponent in the Scobey equation, the magnitude of the coefficient α was found to be 0.63. Accordingly, the Scobey coefficient C_s for PVC pipe with coupler for 6-m longitude is 0.271 (0.43×0.63). For other pipe coupler longitudes, the Scobey coefficient C_s is 0.252 for 9-m longitude and 0.245 for 12-m longitude.

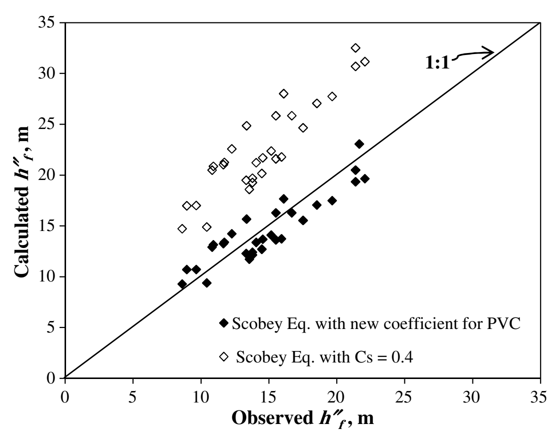

Fig. 9. Calculated versus observed friction head loss by using the proposed coefficient for PVC of the Scobey equation with the modified Christiansen formula

Table 3. Illustrating Examples of Calculating Friction Head Loss and Correction Factors for Aluminum and PVC Pipes

Arithmetic process	Example I: Aluminum pipe	Example II: PVC pipe
Common calculations		
Relative roughness $(\frac{e}{D})$	$\frac{e}{D}=\frac{0.15}{100}=0.0015$	$\frac{e}{D}=\frac{0.03}{152}=0.0002$
Flow velocity (V)	$V=\frac{Q}{A}=\frac{0.020}{\frac{\pi}{4}(0.100)^2}=2.55\ \text{m/s}$	$V=\frac{Q}{A}=\frac{0.050}{\frac{\pi}{4}(0.152)^2}=2.76\ \text{m/s}$
Reynolds number (R)	$\mathsf{R}=\frac{VD}{\gamma}=\frac{2.55\times 0.100}{1.007\times 10^{-6}}=2.5\times 10^5$	$\mathsf{R}=\frac{VD}{\gamma}=\frac{2.76\times 0.152}{1.007\times 10^{-6}}=4.2\times 10^5$
Numerical friction coefficients		
f coefficient	$A=\left\{-2.457\,\text{Ln}\left[\left(\frac{7}{2.5\times 10^5}\right)^{0.9}+0.27(0.0015)\right]\right\}^{16}=2.3\times 10^{20}$	$A=\left\{-2.457\,\text{Ln}\left[\left(\frac{7}{4.2\times 10^5}\right)^{0.9}+0.27(0.0002)\right]\right\}^{16}=4.4\times 10^{21}$
	$B=\left(\frac{37530}{2.5\times 10^5}\right)^{16}=5.5\times 10^{-14}$	$B=\left(\frac{37530}{4.2\times 10^5}\right)^{16}=1.9\times 10^{-17}$
	$f=8\left[\left(\frac{8}{2.5\times 10^5}\right)^{12}+\frac{1}{(2.3\times 10^{20}+5.5\times 10^{-14})^{1.5}}\right]^{\frac{1}{12}}=0.023$	$f=8\left[\left(\frac{8}{4.2\times 10^5}\right)^{12}+\frac{1}{(4.4\times 10^{21}+1.9\times 10^{-17})^{1.5}}\right]^{\frac{1}{12}}=0.016$
C_{HW} coefficient	$C_{\text{HW}}=130$ (From Keller and Bliesner 1990)	—
C_S coefficient	—	$C_S=0.63\times 0.43=0.27$
C_{DW} coefficient; since $e>0.01$, C_{DW} cannot be taken constant value (110) and shall be computed from Eq. (16)	$a=\frac{15.089+1.593(100)}{1+0.03(100)}=43.5$	$a=\frac{15.089+1.593(152)}{1+0.03(152)}=46.1$
	$b=\frac{17.96+0.12(100)}{1+0.011(100)}=14.5$	$b=\frac{17.96+0.12(152)}{1+0.011(152)}=13.8$
	$C_{\text{DW}}=43.5-(14.5)\,\text{Ln}0.15=71$	$C_{\text{DW}}=46.1-(13.8)\,\text{Ln}0.03=94.6$
Modified Christiansen Friction correction factors		
For D-W equation, $m=2$	$F'=\left[\frac{1}{2+1}+\frac{1}{2\times 12}+\frac{\sqrt{2-1}}{6\times 12^2}\right]^{0.567}=0.574$	
For H-W equation, $m=1.852$	$F'=\left[\frac{1}{1.852+1}+\frac{1}{2\times 12}+\frac{\sqrt{1.852-1}}{6\times 12^2}\right]^{0.567}=0.589$	—
For Scobey equation, $m=1.9$	—	$F'=\left[\frac{1}{1.9+1}+\frac{1}{2\times 12}+\frac{\sqrt{1.9-1}}{6\times 12^2}\right]^{0.567}=0.584$
For proposed equation, $m=2$	$F'=\left[\frac{1}{2+1}+\frac{1}{2\times 12}+\frac{\sqrt{2-1}}{6\times 12^2}\right]^{0.567}=0.574$	
h_f calculations		
With D-W equation	$h_f''=0.0826\times 0.574\times 0.023\times\frac{120\times(20\times 10^{-3})^2}{(100\times 10^{-3})^5}=9.02\ \text{m}$	$h_f''=0.0826\times 0.574\times 0.016\times\frac{120\times(50\times 10^{-3})^2}{(152\times 10^{-3})^5}=2.76\ \text{m}$
With H-W equation	$h_f''=1.22\times 10^{10}\times 0.589\times\frac{1}{(130)^{1.852}}\times\frac{120\times(20)^{1.852}}{(100)^{4.871}}=8.28\ \text{m}$	—
With Scobey equation	—	$h_f''=2.05\times 10^{12}\times 0.584\times 0.27\times\frac{120\times(50\times 10^{-3})^{1.9}}{(152)^{4.9}}=2.67\ \text{m}$
With proposed equation	$h_f''=0.0826\times 0.574\times\frac{1}{71}\times\frac{120\times(20\times 10^{-3})^2}{(100\times 10^{-3})^{5.2}}=8.85\ \text{m}$	$h_f''=0.0826\times 0.574\times\frac{1}{94.6}\times\frac{120\times(50\times 10^{-3})^2}{(152\times 10^{-3})^{5.2}}=2.70\ \text{m}$

Fig. 9 shows measured h_f'' versus computed h_f'' by the Scobey equation with C_s coefficients of 0.252, presently developed for PVC pipe, and 0.40, originally developed for aluminum pipe. It is clear from Fig. 9 that values of h_f'', calculated by the Scobey equation with the derived C_s coefficient valid for PVC pipe, agreed with measured h_f'' values.

Illustrating Examples

Two numerical examples are presented to demonstrate the calculation procedure of friction head loss for the D-W equation with f computed from the Churchill equation, for the H-W equation, for the proposed equation, and for the Scobey equation with its derived C_s coefficient suitable for PVC pipe type (Table 3).

The first example is for aluminum pipe with couplers 12 m long, $e = 0.15$ mm, $D = 100$ mm, and $Q = 120$ L/s. The second example is for PVC pipe with couplers 6 m long, $e = 0.03$ mm, $D = 152$ mm, and $Q = 50$ L/s. The other common variables for both pipe types are $N = 12$ outlets, $L = 120$ m, and $\upsilon = 1.007 \times 10^{-6}$ m^2 s. The calculation steps are tabulated as shown in Table 3.

From the first example (I), the calculated relative error for h_f'' computed by the H-W equation compared to h_f'' computed by the D-W equation is -8.2%. On the other hand, the estimated relative error for h_f'' computed by the proposed equation compared to h_f'' computed by the D-W equation is -1.8%. In the second example (II), the estimated relative errors for h_f'' computed by the Scobey and proposed equations compared to h_f'' computed from the D-W equation were -3.5 and -2.1%, respectively. Ultimately, the results of the two numerical examples proved that the H-W coefficient C_{HW} cannot be considered constant with the pipe diameter, and that the derived Scobey coefficient C_s is suitably valid for PVC pipe.

Conclusions

Observations of 30 center pivots were used to assess four different friction head loss, h_f, equations and four frication correction factor, F', formulas. The h_f was calculated by using the D-W, H-W, Scobey, and proposed equations. On the other hand, the F' was computed by using the typical Christiansen, modified Christiansen, Anwar, and Alazba formulas. The ultimately computed friction head loss, h_f'', was compared to the observed values where the evaluation was based on the statistical error techniques, RMSD, NRMSD, ME, OI, and SRM.

Both D-W and H-W equations, along with the proposed equation, showed the friction head loss calculations in good comparison with the field observations when using the modified Christiansen, Anwar, and Alazba formulas. Expectantly, this agreement was not observed for the three h_f mentioned equations when combined with the typical Christiansen formula. It might be worth mentioning that the proposed equation was developed through mathematical simulation and statistical analysis for wide ranges of V, D, and e. The ranges of variables covered smooth and rough pipe types and all turbulent flow types, i.e., $R > 4{,}000$.

The Scobey equation overestimated the determination of friction head loss when compared to the field observations. In other words, the computed friction head loss, when combined with the modified Christiansen, Anwar, and Alazba formulas, was not in agreement with the field observations. On the other hand, an agreement occurred between the observed and the calculated friction head loss values when combined with the typical Christiansen formula. This interesting finding encouraged improvements to the estimation of friction head loss using the Scobey equation by incorporating a Scobey coefficient of retardation, C_s, through analytically mathematical derivation, suitably valid for PVC pipe type. Ultimately, a good match occurred between the observed and calculated h_f'' using the Scobey equation in combination with the modified Christiansen, Anwar, and Alazba formulas. Finally, the proposed and modified Scobey equations, along with any of the frication correction factor formulas, with exception of the typical Christiansen formula, can be used to determine the frication head loss for center pivot with insignificant loss of accuracy.

Notation

The following symbols are used in this paper:

C_{DW} = numerical friction coefficient for proposed friction head loss equation;
C_{HW} = H-W coefficient;
C_S = Scobey coefficient of retardation;
D = inside pipe diameter;
e = equivalent roughness height;
e/D = relative roughness for lateral;
F = correction friction factor for flow decreasing linearly;
f = D-W friction factor;
F' = correction friction factor for flow decreasing non-linearly;
h_f = friction head with one outlet;
h_f' = energy loss due to friction with multiple outlets of linear decline in discharge;
h_f'' = energy loss due to friction with multiple outlets of nonlinear decline in discharge;
i = integer $(2, 3, 4, \ldots, N)$;
j = integer $(1, \ldots, i)$;
L = pipe length;
m = velocity exponent;
N = number of outlets along the lateral;
n = number of observations;
Q = flow rate;
R = dimensionless Reynolds number;
V = flow velocity;
$x_{c,i}$ = calculated value;
$x_{\max}$ = maximum observed value;
$x_{\min}$ = minimum observed value;
$x_{o,i}$ = observed value;
$\bar{x}_o$ = averaged observed values; and
υ = kinematic viscosity.

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